14. LINEAR AND NONLINEAR APPROXIMATION

14.1 General Linear Approximation

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%%% Example 14.1: Approximate values (x(i),y(i)), pro i=1,2,...,N
%%%    by a function f(x)=a(1)*x^3+a(2)*x
% Definition of given values
    x=[0 0.2 0.5 0.7 0.8 1 1.2 1.6 1.9 2]';
    y=[0 0.2 0.6 1.0 1.3 2 2.9 5.7 8.8 10]';
% Definition of the matrix of normal equations and its solution
    A=[x.^3 x]; a=A\y
% Plot of given and approximation values
    xx=0:0.05:2; f=a(1)*xx.^3+a(2)*xx; plot(x,y,'or',xx,f); grid on
      xlabel('x'); ylabel('y'); title('APPROXIMATION');
```

14.2 Nonlinear Approximation and Gradient Method

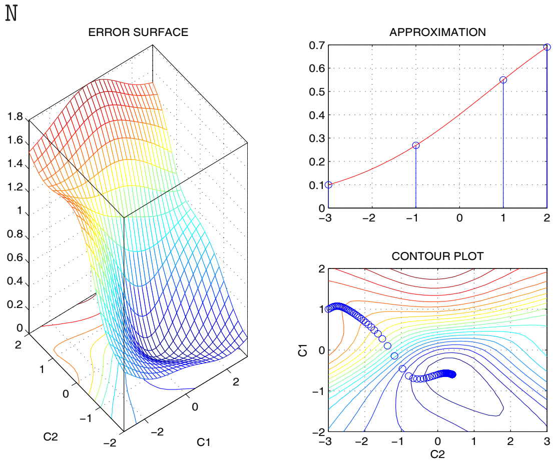
Problem statement: Approximation of given values $\{x(i), y(i)\}_{i=1}^{N}$ by a function $f(\mathbf{a}, x)$ of parameters $\mathbf{a} = \{a(j)\}_{j=1}^{M}$

Solution: 1. Statement of the mean square error evaluation:

$$S(\mathbf{a}) = \sum_{i=1}^{N} (f(\mathbf{a}, x(i)) - y(i))^2 \tag{2}$$

2. Coefficients estimation: coefficients $\mathbf{a}$, α and loop including
3. Gradient evaluation $DA1 = \partial(S)/\partial(a(1)$; $DA2 = \partial(S)/\partial(a(2)$; ...
4. Update of coefficients: $a(1) = a(1) - \alpha * DA1; a(2) = a(2) - \alpha * DA2; \ldots$

```
%%% Example 13.2: Approximate values (x(i),y(i)), pro i=1,2,...,N
%%%    by a function f(x)=f(x)=1/(exp(c1*x+c2)+1)
% Definition of given values and error surface plot
  x=[-3 -1 1 2]'; y=1./(exp(-0.6*x+0.4)+1);
  C1=-2:0.2:2; C2=-3:0.2:3;
  for i=1:length(C1); for j=1:length(C2)
    S(i,j)=sum((y-fa([C1(i),C2(j)],x)).^2); end; end
% Solution by gradient method
  c10(1)=1; c20(1)=-3; alpha=0.8; M=250; plot(c20,c10,'o');
  for k=1:M
    dc1=sum((y-fa([c10(k),c20(k)],x)).*...
      exp(c10(k)*x+c20(k))./(exp(c10(k)*x+c20(k))+1).^2 .*x);
    dc2=sum((y-fa([c10(k),c20(k)],x)).*...
      (exp(c10(k)*x+c20(k)))./(exp(c10(k)*x+c20(k))+1).^2);
    if(abs(dc1)+abs(dc2)<0.000001), break; end;
      c10(k+1)=c10(k)-alpha*dc1; c20(k+1)=c20(k)-alpha*dc2;end;
% Plot of given and approximation values
  subplot(1,2,1); meshc(C2,C1,S); axis tight
  subplot(2,2,4); contour(C2,C1,S,50);
    hold on; plot(c20,c10,'o'); hold off
  subplot(2,2,2); stem(x,y); hold on;
   xx=min(x):0.1:max(x); plot(xx,fa([c10(end), c20(end)],xx),'r'); hold off;
   grid on;
```




```
function f=fa(c,x)
  f=1./(exp(c(1)*x+c(2))+1);
```

COMMANDS

backslash

LSQLIN
LSQNONLIN
FMINSEARCH

LSQCURVEFIT

SOLVE

EXAMPLES 14

14.1 Approximate given sequence
x=[0.3 0.4 0.6 0.9 1.5 2]';
y=[0.4 0.6 1.0 1.7 3.8 6]';
by a function $f(x) = x + a(2) * x^2$ and plot results

15. STATISTICAL DATA ANALYSIS

15.1 Basic statistical characteristics

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%%% Example 14.1: For the set of given values (x(i),y(i)), i=1,2,...,N evaluate
%%%    their mean value, standard deviation, and correlation coefficient

% Definition of given values
    x=[0 0.2 0.5 0.7 0.8 1 1.2 1.6 1.9 2]';
    y=x+0.1*rands(10,1);
% Evaluation
    [mean(x) mean(y)]
    [std(x) std(y)]
    corrcoef(x,y)
```

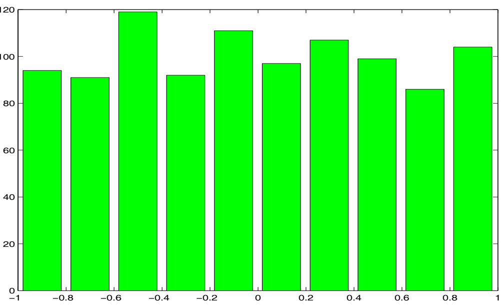
```
%%% Example 15.2: Evaluate and plot the histogram of distribution of
%%% random values rands(1000,1)

    [h,x]=hist(rands(1000,1));
    bar(x,h,'g')
```

COMMANDS

MEAN
STD
CORRCOEF
RANDS

HIST
BAR



EXAMPLES 15

15.1 Evaluate basic statistical characteristics of all columns of matrix $R = rands(100, 5)$

15.2 Analyse distribution of random values generated by function $R = randn(N, 1)$ for a chosen number N of its values